



Characterization of Diameter Distribution and Prediction of Weibull Parameters Equation for Plantation-grown Eucalyptus Species

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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ABSTRACT

Aim: To characterization of the diameter distribution and prediction of Weibull parameters of a plantation-grown Eucalyptus species.

Study design: Stratified sampling method was adopted, in which the plantation was stratified into four age series.

Place and duration of study: Afaka Forest Reserve, one month.

Methodology: Fifty (50) sample plots of 20 x 20 m were laid across the age series. In each of the plot, all the trees were counted and data of variable of interest was collected and processed. A separate Weibull distribution is fitted to the diameter at breast height (dbh) frequency data from each of the plot for the estimation of Weibull parameters (location, scale and shape). The data set obtained from the Weibull parameter estimate was then used to develop regression equations with the stand variables. Coefficient of Determination (R^2) and Root Mean Square Error (RMSE) was used as goodness of fit test.

Results: The result on the stand characteristics revealed that, the mean diameter at breast height (dbh) ranges between 13.4 – 18.2 cm across the four stands. This indicates that the species are still of pole sizes. The average site productivity of the species ranges between 24.0 m to 37 m at an index of 25 years. The mean Basal area varies between 14.13 to 26.85 m² per ha, while the

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average tree total height ranges between 24.6 to 28.2 m across the four species. The result on diameter class distribution shows that most of the species fell within dbh class of 11 -20 cm class except *E. cloeziana* in which the highest frequency fell into 16 – 20cm dbh class. Best equation were selected for each of the Weibull parameters (α , β , γ) per species based on fit statistics. The formation of straight line pattern from the plotted normal probability plots indicates the adequacy of the selected models for predicting Weibull parameters. A fluctuation pattern exists between the Weibull parameters and the stand characteristics. this may be due to variation in climatic factor, most especially fluctuations in rainfall pattern in the area at that particular period.

Conclusion: The ease of fit and high value of coefficient of determination of the models in this study has re-affirmed the use of Weibull parameter in prediction of stand characteristics as been suggested by many authors in the literature.

Keywords: Parameters; distribution; productivity; stratified; fluctuation.

1. INTRODUCTION

Forest managers often require information concerning the size-class distribution of a forest stand, often in the form of a tabulation of numbers of trees by diameter class. This diameter distribution information is often used to predict volume production, i.e. the primary variable that forest managers are interested in. Diameter distribution models are also important in forecasting the range of products that might be expected from a stand. The diameter information of a stand is as important as the information on total volume, although, several distribution models have been in use in which some are flexible enough to describe the growth curves. Tree diameter distributions play an important role in stand modelling. In recent forest practice, several models of probability density of tree diameters in stand have been verified.

There is, for example, great interest in knowing the number of trees in a diameter class in a stand in which a particular silvicultural treatment has been applied, because the diameter sizes determine the industrial use of the wood and thus the price of the different products. Diameter distributions also give information about stand structure, age structure, stand stability, etc. and it enables planning of silvicultural treatments.

Tree diameter distribution is usually characterized by estimating the parameters of some theoretical distribution [1]. Many probability distributions have been used for this purpose, among which are, the normal distribution [2], Lognormal distribution [3], exponential distribution, beta distribution [4], gamma distribution [5] and Johnson SB distribution [6]. The Weibull probability density function was first used for modelling diameter distributions of pure and even-aged stands [7], and since then it has

been used in many growth models based on diameter distributions because of its flexibility and simplicity [8,9,1,10,11].

Since Bailey and Dell introduced the Weibull distribution in forestry in 1973, many researchers considered Weibull as one of the best-performing distributions for modelling diameter distributions [12]. [13] and [14] credit the extensive use of Weibull distribution due to its flexibility and capacity for fitting a variety of shapes. In addition, the Weibull also has the advantage of showing a good correlation of its parameters with stand attributes [7], such as dominant height, quadratic mean diameter, mean diameter, and its percentiles, among others [15].

Several studies have reported the use of Weibull function to describe the size distribution of trees and stands species. e.g. Douglas fir [16], eastern cottonwood [17], Scots pine [18,19], black spruce [20], slash pine [21], loblolly pine [22,23].

In Nigeria, different growth models based on diameter distribution have been developed, for species like *Tectonal grandis* [24], *Nauclea diderichii* [25], *Gmelina aborea* [26], *Pinus patula* [27] etc. but there is little or no information on the diameter distribution of the introduced exotic species (eucalyptus), the specie in the northern part of Nigeria. *Eucalyptus*, being one of the worlds' most important specie is used for timber production due to its hardness and resistant's to termite's attacks while others which are soft are used for pulping in paper production. Thus, for future requirements of sawn logs and industrial wood to be met, there is need for adequate information on the diameter distribution of the species.

With all the above mentioned qualities possessed by eucalyptus species, and to bridge

the gap between crude stand-level simplifications and complex individual tree models, size distribution models are potent tools for providing more detailed knowledge on the forest structure, product value, and forest operations costs for forest managers and researchers, without additional inventory costs. The focus of this study is to characterize the diameter distribution and prediction of Weibull parameters equation as a suitable parameter for plantation-grown eucalyptus species in Afaka forest reserve.

2. MATERIALS AND METHODS

2.1 Study Area

The study was carried out in the FRIN/JICA *Eucalyptus* plantation site, located in Afaka Forest Reserve, Igabi Local Government Area, Kaduna State (Fig. 1). The plantation lies along Latitude ($9^{\circ} 00''$ and $11^{\circ}30''N$) and Longitude ($6^{\circ}30''$ and $8^{\circ}30''E$) and covers up to 2,700 hectares extending in a Southern-Easterly direction within the reserve of almost ten thousand hectares (10,000 ha). Kaduna State, experiences a typical tropical continental climate with distinct seasonal regimes, oscillating between cool to hot dry, and humid to wet.

Generally, the soils and vegetation are typical red-brown to red yellow tropical ferruginous soils and savannah grassland with scattered trees and woody shrubs. The soils in the upland areas are rich in red clay and sand, but poor in organic matter. However, soils within the "Fadama" areas are richer in kaolinitic clay and organic matter, very heavy and poorly drained, characteristics of vertisols. The bedrock geology is predominantly metamorphic rocks of the Nigerian Basement complex, consisting of biotite, gneisses and older granites. In the southeastern corner, younger granites and batholiths are evident. Deep chemical weathering and fluvial erosion, influenced by the bioclimatic nature of the environment, have developed the characteristic high undulating plains with subdued interfluves.

2.2 Methods of Data Collection

Data for this study were collected in the fifty sampled plots of (20 x 20m) in size, established across the species four age series. Tree variable of interest were collected in each of the sampled plots, processed and estimated. The data collected from tree measurement was processed into a suitable form for statistical analysis and the following variables were estimated.

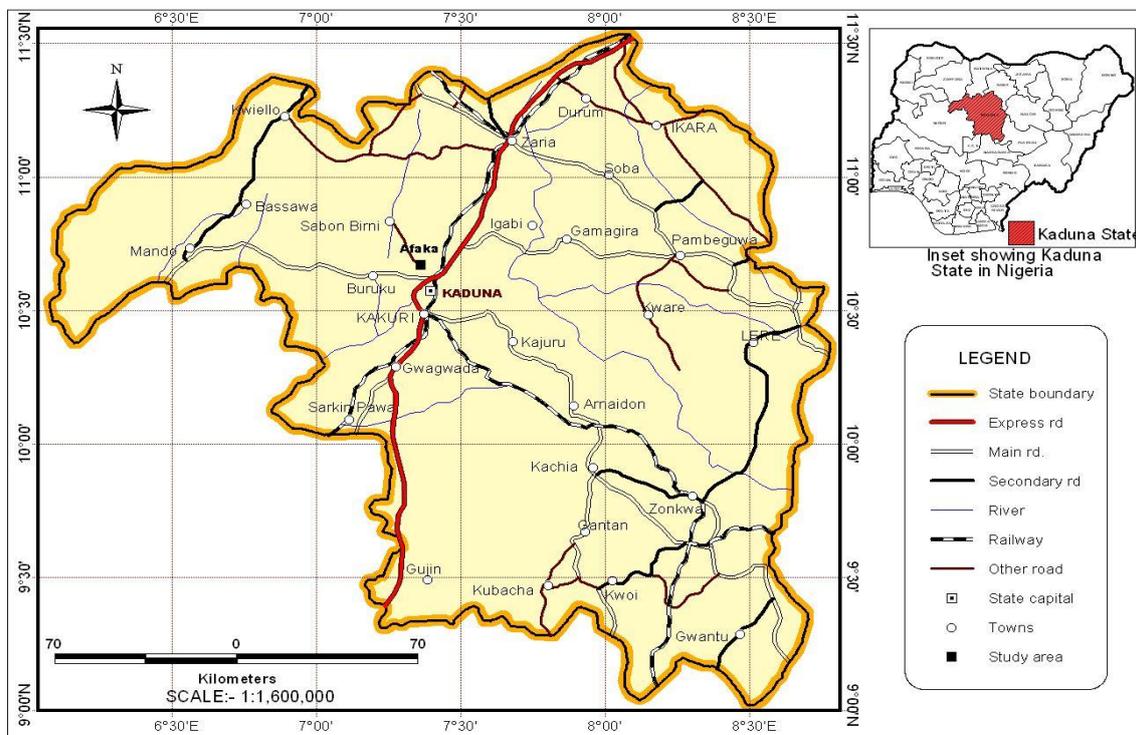


Fig. 1. Map of Kaduna State showing the study area (Afaka)
 Source: GIS Unit, College of Forestry and Agricultural Mechanization, Afaka

2.3 Weibull Parameters Estimates per Plot

The moment estimation procedure, as described by [28] and [29] was adopted for this study, to estimate the percentile (Equation 3) for the three-parameter Weibull distribution, which was based on the 24th, 63rd, and 93rd percentiles. The estimated percentile were then equated to its corresponding Weibull distribution function, which was then solved iteratively for estimates of α , β , and γ for each of the sampled plot, using equation 1.

$$X_p = \alpha + \beta[-\ln(1 - p)]^{1/\gamma} \quad [1]$$

Using the 24th, 63rd, and 93rd percentiles, equation (2 - 4) produced the following set of simultaneous equations:

$$X_{0.24} = \alpha + \beta[-\ln(0.76)]^{1/\gamma} \quad [2]$$

$$X_{0.93} = \alpha + \beta[-\ln(0.07)]^{1/\gamma} \quad [3]$$

$$X_{0.63} = \alpha + \beta \quad [4]$$

Re-arrangement of equation 2– 4, produces equation 5:

$$\frac{X_{.24} - X_{.63}}{X_{.93} - X_{.63}} = \frac{[-\ln(.76)]^{1/c} - 1}{[-\ln(.07)]^{1/c} - 1} \quad [5]$$

$$\beta = \frac{(X_{.24} - X_{.63})}{\left\{ \frac{1}{[-\ln(.76)]^{1/c} - 1} \right\}} \quad [6]$$

and

$$\alpha = X_{.63} - \beta \quad [7]$$

For any given set of values for $X_{.24}$, $X_{.63}$ and $X_{.93}$, equation (5) was solve iteratively to obtain shape parameter (γ), which was then used in equation (6) to produce β parameter estimate. The estimated scale parameter was then used to determined the estimate of the origin parameter (α) in equation (7).

The diameter distribution at each plot was described by the three-parameter Weibull probability density function, and it is of the form:

$$f(X) = \frac{\gamma}{\beta} \left(\frac{X-\alpha}{\beta} \right)^{\gamma-1} \exp \left[- \left(\frac{X-\alpha}{\beta} \right)^\gamma \right] \quad [8]$$

if, $\alpha < X < \infty$

if $X < \alpha$

The ' β ' and γ parameters must always be positive, while ' α ' can be positive, zero or negative, but for its application for diameter distribution, ' α ' must be non-negative. The parameters estimates were obtained via Maximum Likelihood Estimator (MLE), using SPSS Version 17.0.

2.4 Regression Equation for Weibull Parameters

The data set obtained from the Weibull parameters estimates (Table 1) were used to develop a regression equation, using stand variables as the explanatory variables.

2.5 Model Selection Criteria

The selected models for each of the Weibull parameters were evaluated, using:

1. A goodness of Fit with high coefficient of determination (R^2) and

$$R^2 = 1 - \frac{\sum_{i=1}^m \sum_{j=1}^n (W_{ij} - \widehat{W}_{ij})^2}{\sum_{i=1}^m \sum_{j=1}^n (W_{ij} - \overline{W}_{ij})^2} \quad [9]$$

2. Least Root Mean Square Error (RMSE).

$$RMSE = \sqrt{\frac{\sum_{i=1}^m \sum_{j=1}^n (W_{ij} - \widehat{W}_{ij})^2}{n-p}} \quad [10]$$

Where:

W and \widehat{W} = the observed and predicted values of Weibull parameters

m and n = the number of plots and number of observations used to fit the model respectively, while

p = number of model parameters.

3. RESULTS AND DISCUSSION

The summary statistics of the stand attributes are presented in Table 1. The result revealed that the unthinned plantation has an average stocking of 897 stems per hectare (80.4%). This indicates little or no encroachment in the plantation, since 1,111 stems per hectare can still be obtained with an espacement of 3 x 3 m in the stands.

The result on the mean tree density revealed a total of 925, 923 and 906 stem per hectare for *Eucalyptus citriodora*, *E. cloeziana* and *E.*

camaldulensis stands respectively, while, *Eucalyptus tereticornis* had the least (831 stems/ha). The mean diameter at breast height (DBH) ranges between 13.4 – 18.2 cm across the four stands (Fig. 2). This indicates that the species are still of pole sizes. The average site productivity of the species ranges between 24.0 m to 37 m at an index of 25 years. The mean Basal area varies between 14.13 to 26.85 m² per ha, while the average tree total height ranges between 24.6 to 28.2 m across the four species.

The visual interpretation of tree frequency against diameter class distribution is depicted in fig. 2, with *E. camaldulensis*, *E. tereticornis* and *E. citriodora* having their highest tree frequencies fell into 11 -15cm dbh class (Fig. 2a, b, and c), while, that of *E. cloeziana* fell into 16 -20cm dbh class (Fig. 2d).

3.1 Weibull Parameters

The results on the Weibull parameters estimate and the stands characteristics are presented In Table 2. As expected, none of the parameters showed a negative value. This is in line with the results of [30], who reported that 'β' and 'γ' parameter should always be positive and that even though 'α' parameter can generally be positive, zero or negative, but for diameter distribution, it must not be negative. The Weibull parameter estimates derived from the selected models were presented in Table 3.

This result is in line with the work of [23] who reported that the use of cumulative distribution function Regression was best for predicting Weibull parameters to characterize current stands with known stand attributes. In this study, the CDF Regression was also found to be the most appropriate method for a future stand where its attributes had to be predicted from current attributes. Also, [31] used regression analysis to establish the relationship between Weibull parameters and stand basal area, number of trees per hectare and elevation of the site.

3.2 Diameter Distribution

Diameter at breast height (DBH, 1.3 m above ground level) is the explanatory variable most commonly used in single- and multiple-entry equations to predict tree-level attributes, mainly because it is easy to measure in the field and is

strongly related to many forest variables [32]. The shape of the diameter distribution is one of important elements characterizing forest structure. In this study, a percentile method was used for the estimation of a 3-parameter Weibull function. The result revealed a fluctuations pattern in the diameter distribution of the species and the plantation age. This may be due to variation in climatic factor, most especially fluctuations in rainfall pattern in the area at that particular period. The parameters of the Weibull functions are simply estimated from the basic stand level variables namely basal area, age, number of stem per hectare. The approximate value of the coefficient of determination (R^2) obtained for the location parameter (α), the scale parameters (β) and the shape parameter (γ) across the four species ranges between 0.701 and 0.999. The high R^2 value exhibited by the three Weibull parameters indicates the goodness of percentile methods of estimating Weibull parameters. The results obtained in this study are consistent with those of [33], who obtained more accurate fit with nonlinear regression method than with the product-moment for *Pinus pinea* stands in Valladolid (Spain). The results of this study provide continue support for use of the Weibull probability distribution function (pdf) in describing diameter structures, particularly within mono-specific even-aged stands.

3.3 Regression Equation for Weibull Parameters

Presented in Table 3, is the predicted equations, approximate multiple coefficient of determination (R^2) and corresponding Root Mean Square Error (RMSE) of the regression equations. All equations presented acceptable values of coefficient of determination, indicating that the independent variables used in equations have great influence on the dependent variables. The use of stand variables in predicting Weibull parameters in this study is in line with [31] who used regression analysis to establish the relationship between Weibull parameters and stand basal area, number of trees per hectare and elevation of the site.

Model I with high R^2 and lower RMSE value of 99.3 and 0.2074 respectively was selected as the best model for the location parameter 'α' for *E. camaldulensis*. The explanatory variable (stand basal area and stem per hectare) explain almost 99% of the origin parameter.

Table 1. Summary statistics of eucalyptus species at Afaka forest reserve, Kaduna

Stand Attributes	<i>Eucalyptus camaldulensis</i>				<i>Eucalyptus terreticornis</i>				<i>Eucalyptus citriodora</i>				<i>Eucalyptus cloeziana</i>			
	Mean	Min	Max	Sd	Mean	Min	Max	Sd	Mean	Min	Max	Sd	Mean	Min	Max	Sd
Age(yrs)	25.8	24.0	27.0	1.668	25.7	24.0	27.0	1.078	25.6	24.0	27.0	1.333	25.4	24.0	27.0	0.961
Diameter at breast ht (cm)	13.4	9.9	16.9	2.059	13.9	10.0	17.9	2.165	14.9	12.9	16.9	1.176	18.2	12.9	23.3	3.038
Total height (m)	28.2	20.2	35.1	4.003	23.6	19.1	31.0	3.352	24.1	21.1	27.2	1.825	26.6	23.1	29.6	1.772
Merchantable height (m)	20.9	14.2	26.6	3.356	16.5	13.1	23.4	2.643	17.5	15.4	19.2	1.339	18.8	15.8	20.8	1.315
Dominant height (m)	31.2	27.2	33.7	2.075	29.1	23.5	35.6	1.5988	28.7	26.9	30.3	1.0989	27.3	25.7	29.2	10188
No. of Trees per Compartments	36	30	42	4.002	33	30	37	1.635	37	31	41	3.082	39	34	42	2.183
Stem per hectare (N/ha)	907	750	1050	100.057	831	750	925	49.582	925	775	1025	77.055	923	850	1050	48.501
Basal area per hectare (m ² /ha)	14.13	7.78	20.06	3.719	14.08	7.98	21.88	4.146	17.88	14.38	20.15	2.162	26.85	13.10	45.15	8.598
Site Index (m)	28.9	33.6	36.7	3.0511	30.7	30	32	0.962	30.8	30	32	1.221	30.9	30	32	0.880

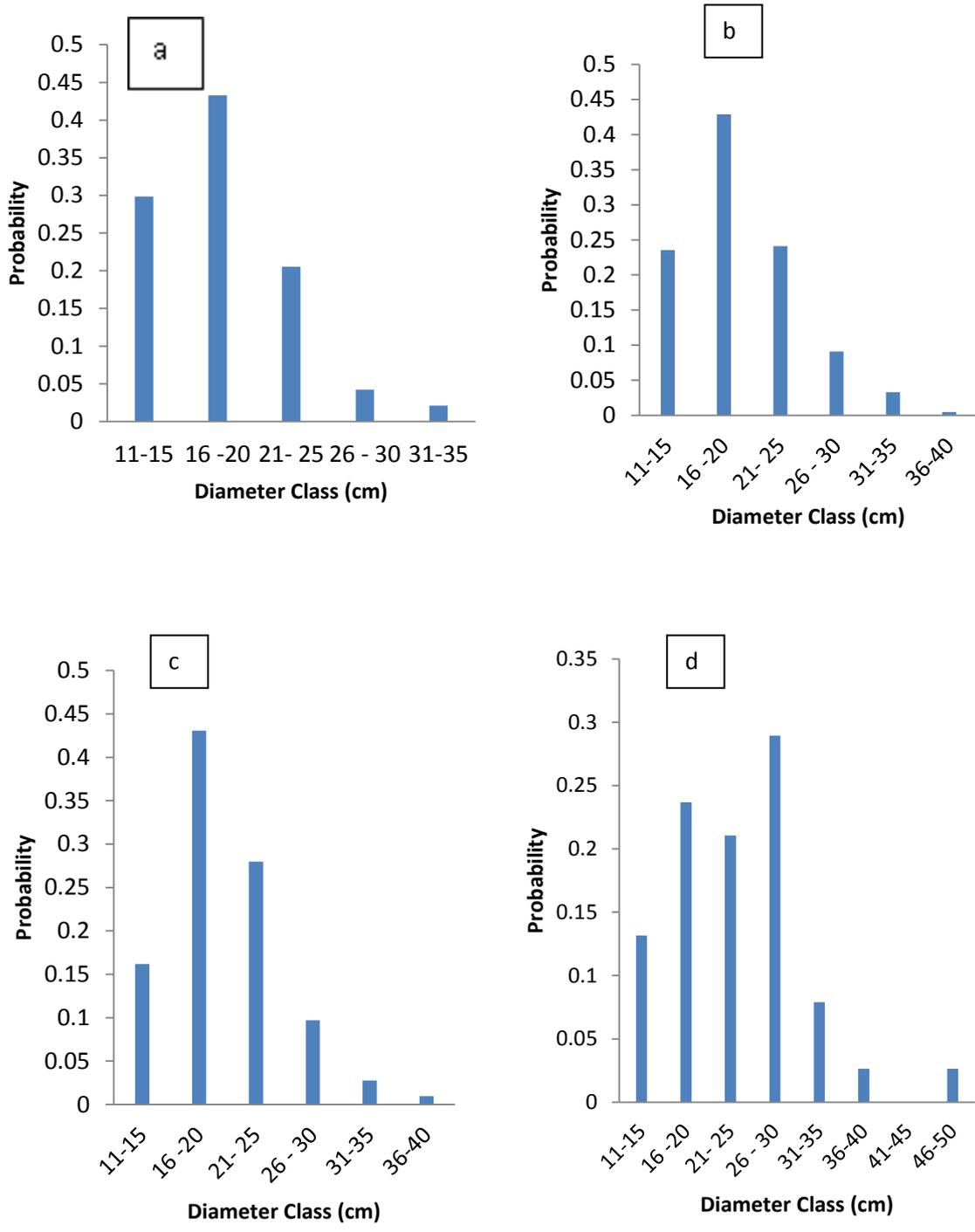


Fig. 2. Diameter Distribution of Eucalyptus species at Afaka Forest Reserve
 (a: *E. camaldulensis*, b: *E. tereticornis*, c: *E. citriodora*, d: *E. cloeziana*)

Table 2. Stand Variable and Weibull Parameters for Eucalyptus species at Afaka forest reserve

Species	<i>E. camaldulensis</i>				<i>E. terreticornis</i>				<i>E. citriodora</i>				<i>E. cloeziana</i>			
Stand Variable																
A (Yrs)	20	21	22	23	20	21	22	23	20	21	22	23	20	21	22	23
N (ha)	975	975	850	881	838	825	858	820	950	842	975	975	967	925	900	950
BA (m ² /ha)	10.76	14.99	13.49	15.68	18.81	16.53	10.72	11.27	15.76	18.62	19.10	19.34	15.35	35.70	27.64	29.30
S.I	29	31	34	37	24	26	28	30	24	27	30	33	26	28	30	32
Weibull Parameters																
α	6.294	9.238	7.944	9.530	9.140	8.669	10.444	8.707	21.729	15.818	19.966	19.996	8.239	14.786	11.446	11.488
B	5.999	6.241	8.477	7.715	5.304	6.556	5.903	9.409	5.784	7.354	4.464	4.417	7.488	7.799	7.980	10.529
γ	1.022	1.327	1.146	1.278	1.184	1.197	1.309	1.242	0.767	0.773	1.020	1.020	1.520	1.677	1.448	1.675

A = Age; Number of trees; BA = Basal area and S.I = Site index; α = Location parameter; β = Scale parameter and γ = Shape parameter

Table 3. Regression equations for the estimated weibull parameters

Species	Model	Parameters	Equations	R ²	RMSE
<i>E. camaldulensis</i>	I	α	$-3.415 + 0.696BA + 0.002N$	0.993	0.2074
	II	β	$63.952 - 0.348BA - 1.714SI$	0.969	0.3768
	III	γ	$1.187 - 0.085BA - 0.05SI$	0.939	0.0632
	IV	α	$-27.530 + 0.045N - 0.050BA$	0.999	0.0548
<i>E. terreticornis</i>	V	Log β	$-3.629 + 0.181A + 0.039BA$	0.972	0.0316
	VI	γ	$2.731 - 0.051A - 0.028BA$	0.880	0.0316
	VII	Log α	$1.313 + 0.001N - 29.631SI^{-1}$	0.817	0.0472
<i>E. citriodora</i>	VIII	β	$21.185 - 0.021 + 71.700BA^{-1}$	0.882	0.8826
	IX	γ	$3.464 - 0.114N + 0.001SI$	0.931	0.0548
	X	α	$13.597 - 0.535A + 0.348BA$	0.998	0.2168
<i>E. cloeziana</i>	XI	β	$-243.886 + 57.345\ln(A) + .487SI$	0.710	1.5502
	XII	γ	$-2.594 + 0.004N + 0.016BA$	0.960	0.0447

BA = Basal area; N = Number of trees; SI = Site index; A = Tree Age

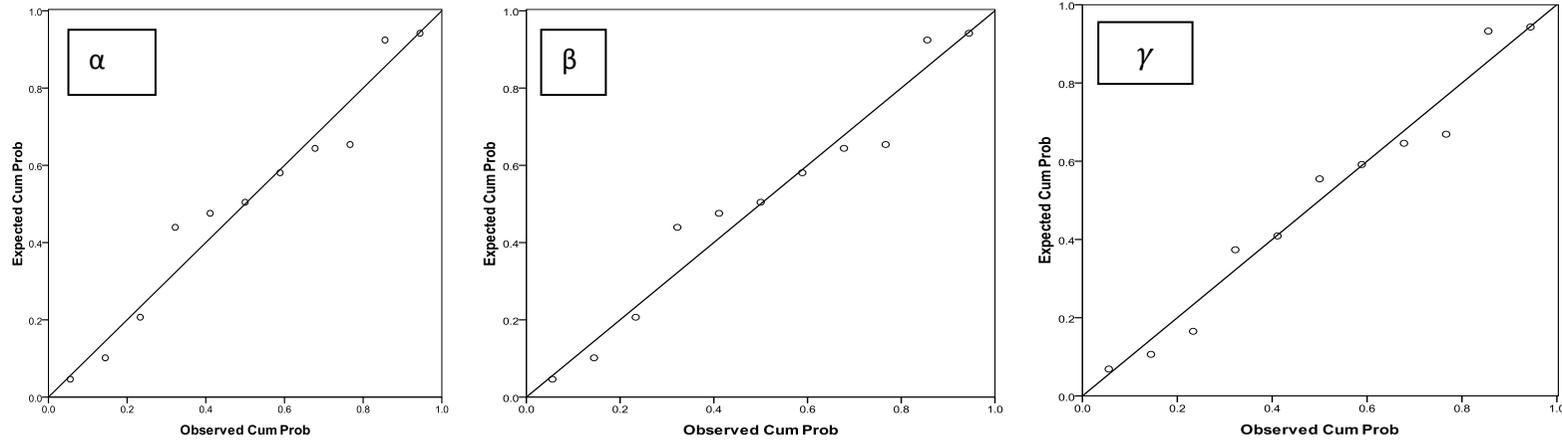


Fig. 3. Normal Probability plot for Weibull parameter for ' α ', ' β ' and ' γ ' for *Eucalyptus camaldulensis*

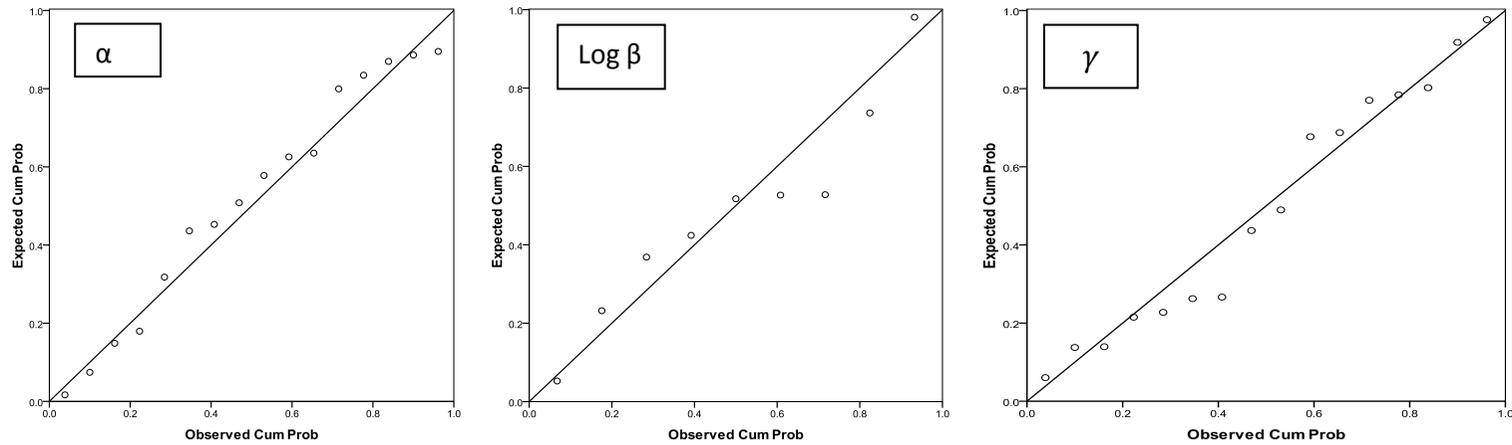


Fig. 4. Normal Probability plot for Weibull parameter for ' α ', ' β ' and ' γ ' for *Eucalyptus tereticornis*

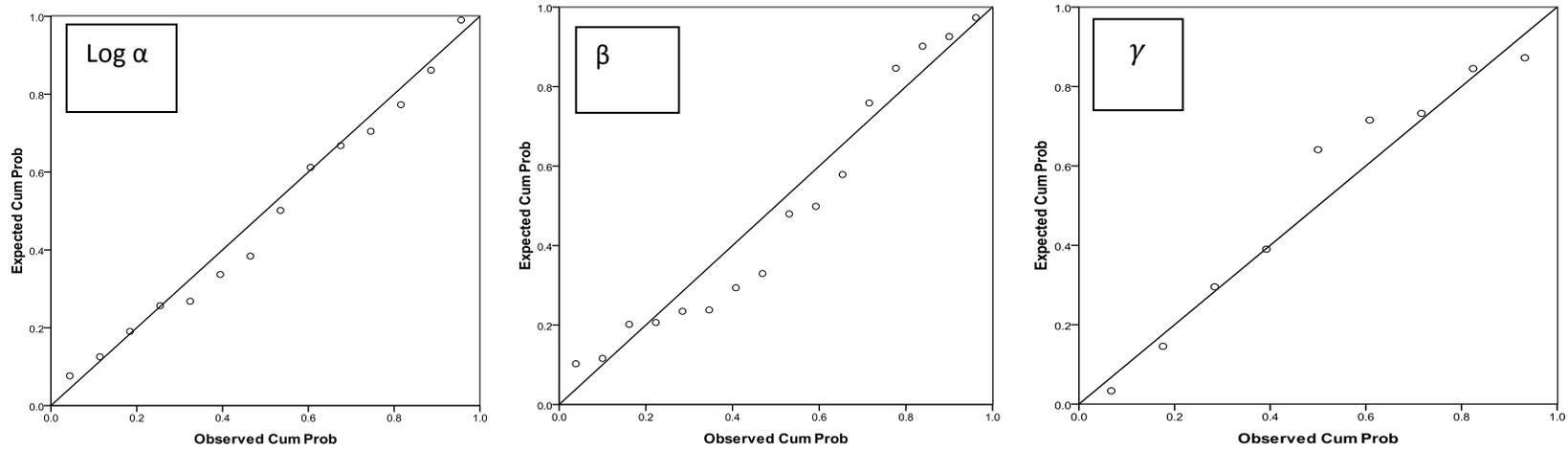


Fig. 5. Normal Probability plot for Weibull parameter for ' α ', ' β ' and ' γ ' for *Eucalyptus citriodora*

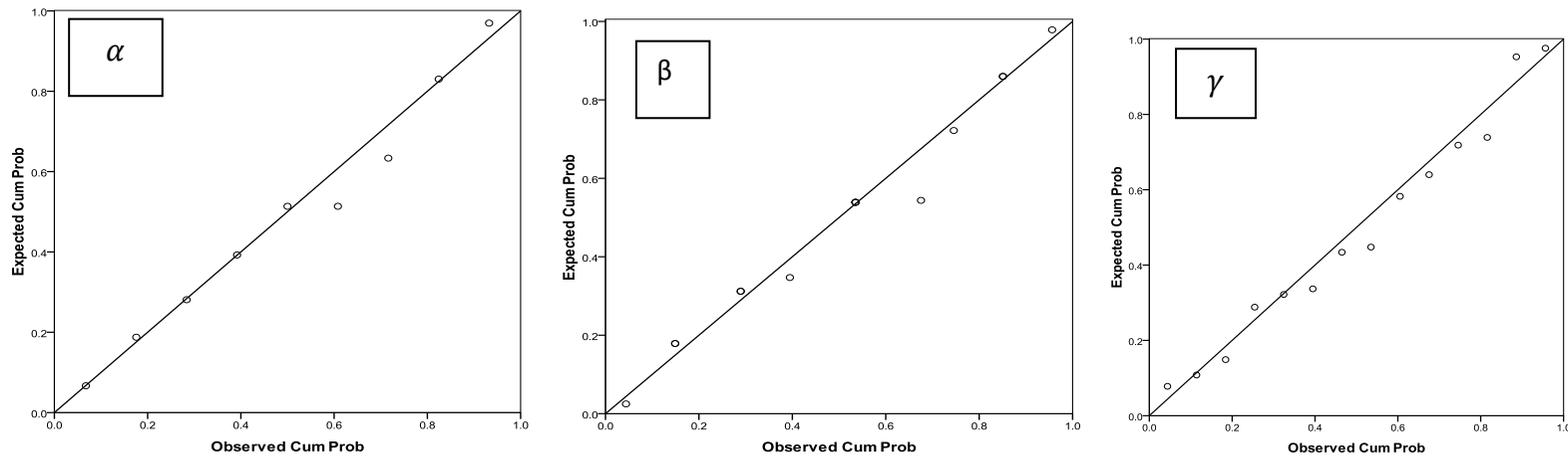


Fig. 6. Normal Probability plot for Weibull parameter for ' α ', ' β ' and ' γ ' for *Eucalyptus cloeziana*

For the scale and shape parameters, model II and III were selected as the best model. The coefficient of determination and root mean square error value (value in parenthesis) of the selected models are 96.9 (0.3768) and 93.9 (0.0632) respectively. The normal probability plots for the selected model parameters for four *Eucalyptus* species are depicted in Figs. 3 - 6. The formation of straight line pattern indicates the adequacy of the selected models in predicting Weibull parameters.

For the location parameter, model IV with high R^2 and lower RMSE value of 99.9 and 0.0548 respectively was selected. The stand basal area per hectare and stand age (independent variables) explains almost 93.9% of the logarithm of the scale parameter for *Eucalyptus terreticornis*; this indicates that age and basal area are very relevant to the prediction of scale parameter. Model V with high R^2 value and RMSE value of 88.0 and 0.0316 respectively was selected as the best model for the shape parameter. The normal probability plot for the selected model is shown in Fig. 4.

For the *Eucalyptus citriodora*, model VII and VIII with R^2 value of 81.7 and 88.2 respectively were selected for the location (α) and scale parameters (β). This implies that, the independent variable (stem per hectare and inverse of site index) explain about 88.2% variation of the location parameter, while number of stem per hectare and age inverse explain almost 74% of the logarithm of the scale parameter. Model IX was selected for the shape parameter (γ). The number of stem per hectare and site index explains about 93.1% of the shape parameters respectively). The adequacy of the selected models was checked by plotting normal probability plot (Fig. 5.).

Model X was selected as the best model for predicting location parameter for *Eucalyptus cloeziana*, the model explains about 99.8% of the origin parameter. For the scale and shape parameters, model XI and XII with high R^2 value of 71.0 and 96.0 and RMSE value of 1.5502 and 0.0447 were selected respectively as the best models. The normal probability plot for the selected model is shown in Fig. 6.

4. CONCLUSION

Diameter distribution of eucalyptus species were characterized, Weibull parameters was estimated and predicted, using stand variables. The result

show that almost three out of the four studied species were deviated towards lower diameter class (16 -20 cm) i.e. below the diameter girth limit. The goodness of fit test of the predicted model showed that the diameter distribution can be accurately estimated, since most of the models gave a high value of coefficient of determination. The ease of fit and high value of coefficient of determination of the models in this study has re-affirmed the use of Weibull parameter in prediction of stand characteristics as been suggested by many authors in the literature. It is flexible and relatively simple to apply to any even-aged forest data.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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